ANALYTICAL COMPUTATION OF MAGNETIZING INDUCTANCES FOR THE SYNCHRONOUS RELUCTANCE MOTOR WITH AXIALLY-LAMINATED ROTOR

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1. INTRODUCTION

The performances of the synchronous reluctance motor are essentially dependent on the magnetic dissymetry of the machine, respectively of its rotor, as in [1], [2], [3], [4], [5], [6]. The last one is made from grain-oriented steel laminations, alternating with insulation and non-magnetic material. The fastening is realized with clamping plates and screws made from non-magnetic material (brass or non-magnetic stainless-steel), to avoid the increase of the q-axis reluctance. Both width of the clamping plates and domain occupied of screws depends on the rotor dimensions and on the number of pole pairs.

In Fig. 1 are shown two examples of rotor shapes for the two-pole machine.

![Example of rotor shape for a two-pole machine](image)

1. laminations; 2. insulation sheet 3. clamping plates
4. non-magnetic stainless steel bolt

The paper presents an analytical method for the air gap flux density and magnetizing inductance estimation. Using the program based on this method one can analyze any axially rotor shape having more than 3 lamination and more than 2 insulation sheets.

In order to find the appropriate rotor shape which can increase the motor performance, an iterative method for a quickly estimation of the magnetizing inductance is very useful.

2. SIMPLIFYING HYPOTHESIS

One agrees that in all the cross sections of the machine the magnetic field is the same; one neglects the magnetic field, which closes through the front area.

One considers the m.m.f. with sinusoidal distribution on the inner surface of the stator.

The influence of stator slots is taken into account by the Carter factor; the influences of the insulation sheets and those of the clamping plates on the equivalent air gap length are considered, as in [7].

One neglects the presence of the non-magnetic stainless-steel bolts.

One considers the non-linearity of the steel laminations only for the d-axis field computation.

One considers that the q-axis field between two adjacent steel laminations of the rotor is a plan-parallel one.
3. THE CALCULATION OF THE D-AXIS AIR GAP FLUX DENSITY

Considering the machine data known (geometrical dimensions, B-H curve of the ferromagnetic material) and the stator m.m.f. distribution on the inner stator surface, one can obtain the d-axis flux density distribution in the air gap, \( B_{\delta d}(x) \). Fig. 2 presents the approximate field line, in order to compute the m.m.f. in the case of a two-pole machine. The length of the approximate field line is calculated as function of the geometrical data and of the number of pole pair. For any field line, the m.m.f. of the magnetic circuit section (air gap, stator yoke, stator tooth and ferromagnetic rotor area) depends on the value of the flux density in the air gap point M, crossed by the field line considered. The coordinate x defines the position of the point M relating to the d-axis.

One divide the motor air gap into sections where the flux density is considered the same, having the value corresponding to the point in the middle of the section.

The number of points where the flux density is computed depends on the difference accepted between \( B_{\delta d} \) values for two adjacent sections. For lamination width less than 1 mm one can consider a single point straight to each lamination. For the considered point one gives as initial value the \( B_{\delta d}(x) \) that corresponds to the case \( \mu_{fe} \rightarrow \infty \).

\[
B_{\delta d}(x) = \frac{\theta_{d}(x)}{\delta_x} \mu_0
\]  

With this value one computes the magneto-motive force for each section of the magnetic circuit. The computed resultant m.m.f. considering \( \mu_{fe} \) finite, became greater than \( \theta (x) \) initially considered and then a new value is given for the air gap flux density. The iterative process continues until the difference between the computed value of the m.m.f. and the initial admitted value is smaller than the fixed error.

In the first air gap area (straight to the rotor part composed of lamination and insulation sheets), one considers a single point straight to each lamination for flux density calculus, because the lamination width is usually less than 1 mm. As result, for this air gap area one computes the flux density for a number of points equal to the lamination number, \( n \).

In the second air gap area (straight to the clamping plates), the flux density obtains a low value (because of the high value of the air gap width, due to the clamping plates), so it is not affected by the non-linearity of the ferromagnetic material. Therefore, one computes the flux density for \( n_c \) points, \( n_c \) being less than \( n \).

Figure 3 presents the d-axis flux density distribution along the half pole pitch (between d and q axis of the rotor) for two cases: considering the non-linearity of the ferromagnetic circuit and considering \( \mu_{fe} \rightarrow \infty \).

The motor has the following data:

\( P = 0.55kW, \ p = 1, \ I = 1A, \ 59 \) rotor insulation sheets, 0.22mm width of the insulation sheets and 0.5mm width of the rotor lamination.

The amplitude of the fundamental of the d-axis flux density in the air gap is obtained from the following equation:

\[
B_{\delta d1} = \frac{4}{\pi} \sum_{\lambda=1}^{n} B_{\delta \lambda} \left( \sin \frac{x_{\lambda f}}{\tau} \pi - \sin \frac{x_{\lambda i}}{\tau} \pi \right) + \sum_{k=1}^{n_c} B_{\delta k} \left( \sin \frac{x_{k f}}{\tau} \pi - \sin \frac{x_{k i}}{\tau} \pi \right)
\]  

4. THE CALCULATION OF THE Q-AXIS AIR GAP FLUX DENSITY

The q-axis air gap flux density is computed considering the magnetic circuit law and the magnetic flux law. The variable \( \lambda \) defines the position of the lamination relating to the d-axis of the rotor.

The Fig. 4 presents the rotor laminations \( \lambda \) and \( \lambda+1 \). The insulation sheet width is \( \Delta_{x,\lambda+1} \).
The q-axis air gap flux density $B_{\delta q}(x)$ straight to the $\lambda$ lamination is:

$$B_{\delta q}(x) = \mu_0 \left( \frac{\theta_q(x) - V_\lambda}{\delta_\lambda} \right)$$  \hspace{1cm} (3)

The q-axis flux which cross any lamination $\lambda$ ($\lambda = 1, 2, \ldots, n-1$) has two terms: the q-axis flux $\Phi_{\lambda+1}$, penetrating from lamination $\lambda+1$ through the insulation sheet, and the q-axis flux which penetrates from the air gap (through the area limited by the coordinates $x_{\lambda i}$ and $x_{\lambda f}$).

$$\Phi_\lambda = \Phi_{\lambda+1} + 2\mu_0 \int_{x_{\lambda i}}^{x_{\lambda f}} \left( \frac{\theta_q(x) - V_\lambda}{\delta_\lambda} \right) dx \hspace{1cm} (4)$$

Only for $\lambda = n$, the equation of the q-axis flux which penetrate the lamination has a single term:

$$\Phi_n = 2\mu_0 \int_{x_{n i}}^{x_{n f}} \left( \frac{\theta_q(x) - V_n}{\delta_n} \right) dx \hspace{1cm} (5)$$

For each insulation sheet placed between the laminations $\lambda$ and $\lambda+1$ one can write the magnetic circuit law:

$$V_{\lambda+1} - V_\lambda = \Phi_{\lambda+1} \frac{\Delta_{\lambda,\lambda+1}}{S_{\lambda,\lambda+1}} \frac{1}{\mu_0} \hspace{1cm} (6)$$

Writing equations as (4) and (6) for all the $n$ laminations ($\lambda = 1, 2, \ldots, n$), one obtains a system having $2n$ equations and $2n$ unknown quantities: $n$ values for the q-axis flux $\Phi_\lambda$ and $n$ values for the magnetic scalar potential $V_\lambda$. Solving this system one obtains the q-axis flux and the magnetic scalar potential of each lamination depending on the geometrical data, the amplitude of the stator magneto-motive force and the permeability of the air gap. With this values one can estimate the air gap flux density straight to each lamination, that is the q-axis air gap flux density $B_{\delta q}(x)$ distribution along the pole pitch.

The amplitude of the fundamental of the q-axis flux density in the air gap is:

$$B_{\delta q 1} = \mu_0 \left( b_{q1} + b_{q2} + b_{q3} \right) \theta \hspace{1cm} (7)$$

where:
\[
\begin{align*}
\beta_{q1} &= \frac{1}{\tau} \sum_{\lambda=1}^{n} \frac{1}{\delta} (x_{lf} - x_{li}) - \frac{1}{2\pi} \sum_{\lambda=2}^{n} \frac{1}{\delta} \left( \sin \frac{x_{lf}}{\tau} \pi - \sin \frac{x_{li}}{\tau} \pi \right) + \\
&\quad + \frac{4}{\pi} \sum_{\lambda=2}^{n} \frac{1}{\delta} C_{V_{\lambda \theta}} \left( \cos \frac{x_{lf}}{\tau} \pi - \cos \frac{x_{li}}{\tau} \pi \right) + \\
\end{align*}
\]

\[
\beta_{q2} = \frac{2}{\pi r_{\delta 1}} \left[ -\frac{\delta + \delta_{c} - r}{r} \pi (\tau - 2x_{rf}) + \cos \frac{x_{nf}}{\tau} \pi + \frac{(\delta + \delta_{c} - r)^2}{r \sqrt{(2r - \delta - \delta_{c})(\delta + \delta_{c})}} \ln b \right]
\]

\[
\beta_{q3} = \frac{4}{\pi r_{\delta 1}} C_{V_{\delta 1}} \left[ \frac{\pi (\tau - 2x_{nf})}{2\tau} + \frac{\delta + \delta_{c} - r}{\sqrt{(2r - \delta - \delta_{c})(\delta + \delta_{c})}} \ln b \right]
\]

\[
b = \sqrt{\delta + \delta_{c} - \sqrt{2r - \delta - \delta_{c}}} \times \left| \frac{\delta + \delta_{c} - r_{g} \frac{x_{nf}}{\tau} \pi + r + \sqrt{(2r - \delta - \delta_{c})(\delta + \delta_{c})}}{(\delta + \delta_{c} - r_{g} \frac{x_{nf}}{\tau} \pi - \sqrt{(2r - \delta - \delta_{c})(\delta + \delta_{c})})} \right|
\]

The terms \(b_{q2}\), and \(b_{q3}\) depend on the geometrical data and emphasize the clamping plates influence.

Figure 5 presents the q-axis flux density distribution along the half pole pitch (between d and q-axis of the rotor) for the same motor.

![Figure 5. The q-axis flux density distribution along a half pole pitch (between d and q-axis of the rotor)](image)

5. THE CALCULATION OF INDUCTANCES

The magnetic flux and the d-axis magnetizing inductance corresponding to \(B_{d1}\) are computed in (12) and (13):

\[
\Phi_{d1} = \frac{8}{\pi^2} I_{C} \sum_{\lambda=1}^{n} B_{\delta \lambda} \left( \sin \frac{x_{lf}}{\tau} \pi - \sin \frac{x_{li}}{\tau} \pi \right) + \sum_{k=1}^{n_c} B_{\delta k} \left( \sin \frac{x_{lf}}{\tau} \pi - \sin \frac{x_{lf}}{\tau} \pi \right)
\]

\[
L_{md} = \frac{2}{\pi} m(N_{k_{p}})^{2} \Phi_{d1} \frac{1}{\theta}
\]

The q-axis magnetizing inductance corresponding to \(B_{q1}\) is:
Computing $L_{md}$ and $L_{mq}$ for different values of the phase current $I$ one obtain the variation $L_{md}(I)$ and $L_{mq}(I)$.

The paper deals not with the computation of the leakage inductance. Based on this method one had developed a program for the estimation of the air gap flux density and the magnetizing inductance considering known the motor data (geometrical dimensions, winding data, B-H curve of the ferromagnetic material) and the stator phase current. One has used the program to analyze a two-pole machine with axially laminated rotor. All the insulation sheets have the same width, $\Delta z$; the lamination width is $\Delta k$.

The running time of this program is significantly shorter in comparison to the finite element method.

6. EXPERIMENTAL RESULTS

The validation of the inductance is realized using the d.c. decay test [8],[9].

The tested motor has the rated power 0.55kW, the supply frequency 50Hz, the phase voltage 110V, the number of poles 2 and the stator phase resistance 3.8 $\Omega$.

Fig. 6 presents the dependence of d-axis magnetizing inductance, $L_{md}$, on the current $I_d$.

![Fig. 6. Variation of the inductance $L_{md}$, on the current $I_d$.](image)

In Fig. 7 is shown the variation of the q-axis magnetizing inductance $L_{mq}$, on the phase current.

![Fig. 7. Variation of the inductance $L_{mq}$ on the current $I_q$.](image)

One finds a satisfactory correspondence between the computed and measured values.
7. CONCLUSIONS

The paper presents the theoretical determination using an analytical method of the air gap flux density respectively of magnetizing inductance for the synchronous reluctance motor with axially laminated rotor. Experimental results of a 0.55kW tested motor validate the proposed method. The running time of the used program is significantly shorter in comparison to the finite element method. Therefore one can analyze a large combination of the rotor data (width and number of the insulation sheet and of the lamination) in order to obtain the required motor performances.

8. APPENDIX

d_{ext} - outer stator diameter; f - supply frequency; k_b - winding factor; l - length of the stack; m - number of phases; n - number of rotor laminations; n_c - number of points considered for the computation of B_{δδ} straight to the clamping plates; N - number of turns per phase; r - rotor radius; δ - geometrical air gap length; δ_{cl} - width of the clamping plate; δ_{λ} - equivalent air gap length corresponding to the ferromagnetic area λ_c Δλ_{λ,λ+1} - width of insulation sheet placed between the laminations λ and λ+1; μ_0 - vacuum permeability; θ - amplitude of m.m.f.; τ - pole pitch

9. REFERENCES