A SIMPLE APPROACH TO INDUCTION MACHINE PARAMETER ESTIMATION

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Abstract – The paper deals with a simple estimation procedure of the squirrel-cage induction motor parameters, like resistances and inductances, considering the data from the machine nameplate. First is presented the analytical calculation according to the conventional steady-state per-phase equivalent circuit, neglecting the iron-core losses. The magnitude of stator-, air-gap and rotor fluxes, required as references by field-controlled scalar and vector control systems, are also determined. For validation of the identified parameters there are presented two simulation structures containing the motor dynamic d-q model, based on the state equations related to a stator-fixed and to a general oriented reference frame. The simulation results are analyzed using the space-phasor theory.

1. INTRODUCTION

In mathematical modeling for simulation of the motor operation modes it is necessary to know the machine parameters. The scalar or a vector control schemes of the motor usually need also information about parameters and other quantities (like a rated flux value), which are not available in the motor data book, but may be calculated based on them, too.

There are common test procedures in order to determine the motor resistances and inductances. For example the basic three tests are the followings [1]:

a) The “DC test” is used to determine the stator per-phase resistance $R_s$;

b) From the “no-load test” at rated stator frequency and current can be determined the stator inductance (approximate value) and also the power loss due to friction and windage including also core losses;

c) The “blocked-rotor test” is useful to determine the rotor per-phase resistance (corresponding to an equivalent three-phase winding referred to the stator turn numbers) and the sum of the leakage inductances of the stator- and rotor windings (referred also to the stator).

In the above-presented procedure only the dividing of the leakage reactance into two values, corresponding to the stator and rotor one, is undetermined. Generally they are assumed equal, however in some type of induction machines a different ratio is suggested [2].

The parameters determined by test have to be correlated in order to realize the rated data of the motor. Usually for analytical calculations the common steady-state equivalent circuit of the machine is considered, which is based on the conventional time-phasor theory, valid for sine-wave operation mode, only.

2. STEADY-STATE EQUIVALENT CIRCUIT FOR ADJUSTABLE FREQUENCY

The equivalent circuit described by the general equations of the induction motor, physically is based on the space-phasor theory and it is suitable for any running mode of the machine including non-sinusoidal, transient, unbalanced and/or asymmetrical operation [3, 4]. It results in DC two-phase quantities, if it is oriented according to a synchronously rotating axis frame (complex plane).

Considering the stator- and rotor fluxes at constant module and rotating with the same angular speed, another type of equivalent circuit is deduced. The rotating emf-es are represented by voltage sources and not by inductances, as is shown in Figure 1.a). Consequently, in the rotor appears the natural induced voltage, expressed by means of the slip angular speed, furthermore the rotor terminal voltage and resistance are no more divided by the slip. This equivalent circuit represents only the electrical quantities. The magnetic effects appear in a distinguished circuit shown in Figure 1,b). In fact the well-known equivalent circuit was spitted into to ones, corresponding separately to the electric and magnetic phenomena, respectively.

Analytical calculations based on the two types of equivalent circuits yield the same results. In fact the steady-state equations, represented with time-phasors, result as a particular case of the general equations written by means of space phasors, because inherently the space phasors contain also “time information” about the phenomena in the machine, not only “space information”. Mathematically in sinusoidal steady-state operation the space phasors degenerate into the common time phasors.
The steady-state model, based on the equivalent circuits shown in Figure 1, achieved from the general one will be preferred to the conventional model because it has more general character.

![Steady-state electrical (a) and magnetical (b) equivalent circuits defined also for zero frequency](image)

**Fig. 1.** Steady-state electrical (a) and magnetical (b) equivalent circuits defined also for zero frequency

In comparison with the conventional steady-state equivalent circuit, this representation is valid also for zero frequency of the rotating magnetic field and it is suitable directly for calculation of characteristics corresponding to variable feeding frequency of the induction motor. The mechanical torque-speed characteristics for constant (that is controlled) voltage, current and fluxes - the so called *Kloss* type curves - are also expressed by the ratio of the actual \(\Delta \Omega\) and critical \(\Delta \Omega_k\) slip angular speed instead of the corresponding per-unit \(s/s_k\) slip values, because the first ones are defined for zero frequency, too [5].

### 3. Calculations Based on the Motor Nameplate Rated Data

On the nameplate of the squirrel-cage three-phase induction motor of *Type B3-160M* there are written the following rated data:

- 5.5 kW is the useful mechanical power on the shaft \(P_N\) in motor running and \(\cos \varphi = 0.735\) the power factor on the stator side;
- stator voltage 220/380 V\(_{rms}\) and stator current 24.3/14 A\(_{rms}\) for \(\Delta/\) Y connection, which corresponds to 311 V and 19.8 A peak values per phase;
- 720 rpm rotor speed \(n_N\) at 50 Hz \(f_sN\), that means the stator has \(2p=4\) pole pairs, the motor synchronous speed is \(n_{syN} = 750\) rpm, the rated slip \(s_N = 4\%\) and the corresponding rotor-current frequency \(f_rN = 2\) Hz.

The synchronous speed of the rotating magnetic field and rotor speed expressed by means of electrical angle are \(\Omega_0N = 314\) rad/s (i.e. also the stator-current angular frequency) and \(\Omega_{an} = 301\) rad/s;

Using the above data may be calculated the active power absorbed \(P_{abs}\) from the supply is:

\[
P_{abs} = 3U_s I_s \cos \varphi_s
\]

where \(U_s\) and \(I_s\) are both per-phase rms values. So the motor total efficiency, which results from the useful mechanical power on the motor shaft \(P_{sh}\), will be:

\[
\eta_{tot} = \frac{P_{sh}}{P_{abs}}
\]

Above the iron-core losses are neglected. It is also useful to calculate the stator input resistance and inductance, defined as:

\[
R_{eq} = \frac{U_s}{I_s} \cos \varphi_s \quad \text{and} \quad L_{eq} = \frac{U_s}{I_s} \sqrt{1 - \cos^2 \varphi_s}
\]

which correspond to the resultant resistance and inductance, respectively of the electrical equivalent circuit from Figure 1.a.

### 4. Method for Analytical Calculation of the Motor Parameters

If we have no possibility to make any test, for a rapid estimation the machine parameters may be obtained according to an approach procedure, which takes into account only the rated data indicated on the nameplate of the motor. In equations valid for any operation point of the motor, that means not only for the rated conditions, in order to keep their general character, the subscript “N” is omitted, in spite of the fact in the followings these
expressions will be used for the rated values. Considering the total efficiency, calculated before from equation (2), first the partial ones \( \eta_{el} \) and \( \eta_{mec} \), corresponding to the electrical and mechanical losses, will be approached from the following expression:

\[
\eta_{tot} = \eta_{el} \cdot \eta_{mec}
\]  

(4)

The electromagnetic power \( P_{em} \) (transferred through the air-gap) in motor action is calculable as:

\[
P_{em} = \frac{P_{sh}}{\xi_{mec}} \cdot \frac{n_{sy}}{n_{m}} = \frac{P_{sh}}{\xi_{mec}} \cdot \frac{\hat{U}_0}{\hat{U}_m}
\]  

(5)

and result the stator “ohmic” losses \( P_{sJL} \) due to the Joule-Lenz effect, from the power difference:

\[
P_{sJL} = 3R_s I_s^2 = P_{abs} - P_{em}
\]  

(6)

Consequently, the stator resistance \( R_s \) may be calculated considering the rated values. It is also determinable directly using characteristic rated values from the following expression:

\[
R_s = \frac{U_s \cos \phi_s}{I_s} - \frac{P_{sh} \cdot 60 \cdot I_s}{n \cdot N_s \cdot \gamma_p \cdot I_s^2} \cdot \frac{1}{\xi_{mec}}
\]  

(7)

where \( N_s = 3 \) is the number of the stator phases. The above expression can be written also as:

\[
R_s = R_{eq} \cdot [1 - \xi_{eq}/(1 - s)]
\]  

(8)

where the electrical efficiency has to satisfy the following condition:

\[
\eta_{tot} < \eta_{el} < 1 - s
\]  

(9)

Usual values for the motor parameters, as resistances \( R_c \) and \( R_r \), leakage \( (X_s \) and \( X_r \), mutual \( (X_m) \) and/or resultant \( (X_s \) and \( X_r \) reactances at the rated frequency \( f_{sN} \) are given [6, 7, 8, 9, 10]. Consequently, can be estimated the useful inductance

\[
L_m = X_m / \Omega 0N
\]  

(10)

and also the resultant inductances, as follows:

\[
L_s = (1 + \sigma_s) \cdot L_m \quad \text{and} \quad L_r = (1 + \sigma_r) \cdot L_m
\]  

(11)

where the leakage coefficients are

\[
\sigma_s = \frac{L_{sM}}{L_m} = \frac{X_{sM}}{X_m} \quad \text{and} \quad \sigma_r = \frac{L_{rM}}{L_m} = \frac{X_{rM}}{X_m}
\]  

(12)

The stator- and rotor time constants are defined as:

\[
\tau_s = \frac{L_s}{R_s} \quad \text{and} \quad \tau_r = \frac{L_r}{R_r}
\]  

(13)

The resultant leakage coefficient at the first site may be approximated depending on the power factor, only [11, 12]. Then considering the stator- and rotor leakage coefficients equals, results:

\[
\sigma = \frac{1 - \cos \phi_{SN}}{1 + \cos \phi_{SN}} \quad \text{and} \quad \sigma_r = \frac{1}{\sqrt{1 - \sigma}} - 1 \approx \sigma_s
\]  

(14)

The final values for all of them will result a little bit higher.

Considering now the common expression of the mechanical torque on the motor shaft

\[
M_{sh} = \frac{30}{\delta} \cdot \frac{P_{sh}}{n}
\]  

(15)

and the air-gap (electromagnetic) torque expressed, as:

\[
M_{em} = M_{sh} / \xi_{mec}
\]  

(16)

The air-gap torque from the above equation may be also expressed by means of the stator current and machine parameters, based on the equivalent circuits. It results like a simplified Kloss’ formula without any neglecting with respect to Figure 1 [5]:

\[
M_{em} = k_M I_s^2 \cdot \frac{L_m}{1 + \sigma_r} \cdot \frac{s \hat{U}_0 \delta_r}{1 + s \hat{U}_0 \delta_r} = 2M_{ki} \left( \frac{s k_i + s}{s k_i} \right)
\]  

(17)

where the pull-out torque and the critical slip are:
The torque coefficient \( k_M = N_s z_p \) for \( \text{rms} \) values and \( N_s z_p/2 \) for peak values (in the case of the common defined space-phasors) [4].

If \( \sigma_s > \sigma_r \) we can take \( \sigma_s \) a little bit higher (5…10%) with respect to the value obtained before from the second expression of (14) and choosing also \( L_m \), the rotor time constant \( \tau_r \) can be calculated from (17), as follows:

\[
\tau_r = (\bar{e}_i \pm \sqrt{\bar{e}_i^2 - 1})/s\Omega_0
\]

(19)

where \( \bar{e}_i = M_{ki}/M_{em} \).

Using the parameters obtained above, the new value of the resultant leakage coefficient is now calculable, which is corresponding to the equivalent circuits from figure 1 and 2. It is:

\[
\delta = \frac{L_{eq} \cdot (1 + \delta_r) \cdot (1 + a_r^2) - L_m}{L_{eq} \cdot (1 + \delta_r) \cdot (1 + a_r^2) + L_m a_r^2}
\]

(20)

where the equivalent inductance is calculated from the second expression of (3) using the rated quantities of the motor and

\[
a_r = s \dot{U}_0 \delta_r = 2 \delta_r \dot{\delta}_r
\]

(21)

is the per-unit value of the rotor time constant (\( f_r = s f_s \) is the frequency of the rotor currents). Using the new value of \( \sigma_s \), the stator leakage coefficient is also recalculated, as follows:

\[
\delta_s = \frac{1}{(1 - \delta) \cdot (1 + \delta_r)} - 1
\]

(22)

and it will results now \( \sigma_s < \sigma_r \).

The stator- and rotor inductances will be determined based on expressions (11). The rotor resistance and the stator time constant result from expressions (12) and (13), respectively. The rotor current result just simply from the well know expression of the air-gap torque:

\[
I_r = \sqrt{\dot{U}_0 M_{em} \over k_M R_r}
\]

(23)

The rated values of the resultant fluxes are required as references (imposed values) scalar- and vector control schemes of adjustable speed drives, which usually use flux controllers, too [8, 9].

The resultant rotor flux is calculable from the following expression:

\[
\Phi_r = L_r \frac{1}{s \dot{U}_0 \delta_r} I_r
\]

(24)

the resultant air-gap flux is described by:

\[
\psi_m = \sqrt{\psi_r^2 + L_r^2 \delta_r^2}
\]

(25)

and the resultant stator flux obtained from:

\[
\Phi_s = L_s I_s \sqrt{1 + {2 \sigma_s^2 a_r^4 \over 1 + a_r^2}}
\]

(26)

The above expressions are determined also from the same equivalent circuits, (see Figure 1) and they are also defined for zero frequency by substituting \( s \Omega_0 \) with \( \Delta \Omega \), as was mentioned before [5].

5. PARAMETER VALIDATION BY SIMULATION

The validation of the parameters will be made by simulation using a mathematical model of the induction motor valid not only for steady state, but also for transient running. It is constituted of the general equations of the induction machine based on the space-phasor theory.

The most simple simulation structure uses the general-oriented two-phase \( d\alpha-q\lambda \) model as is presented in Figure 2,a. It is equipped with two Vector Analyzers (VA block) for calculation the module of the stator-current and
flux phasors. At the output result also the trigonometrical functions (sine and cosine denoted with \( \sin, \cos \)) of the angle between the positions of the analyzed vector (i.e. \( \psi \), of the stator-current one) with respect to the orientation direct axis (\( \lambda \)). If the input voltage vector direct (\( \text{Real-part} \)) component is equal to the voltage peak value (it is a DC quantity) and the quadrature (\( \text{Imaginary-part} \)) one is considered zero, then the angular speed of the rotating complex plane \( \omega = \Omega = 2 \pi f_s \) is taken equal to the stator-voltage angular frequency, then the model will be stator-voltage oriented and the computed trigonometrical functions will contain also the power factor \( \cos \varphi \).

The approach of the motor electrical and magnetical parameters and quantities was made using the steady-state operation mode. It was started and loaded under rated conditions and it achieves the equivalent circuit, but the validation of them is realized by simulation based on the dynamic model according to the general equations of the induction motor. Then, after 0.3 sec, the motor was step-loaded with the rated shaft torque \( M_{\text{sh}} = 81.05 \text{ N.m} \) (Figure 4), stator current \( I_{\text{sh}} = 19.8 \text{ A} \) (Figure 6), stator flux \( \Psi_{\text{N}} = 0.84 \text{ Wb} \) (Figure 7) and resultant stator flux \( \Psi_{\text{z}} = 0.97 \text{ Wb} \) (Figure 8). The electromagnetic torque after the transients becomes equal to the sum \( M_{\text{sh}} + M_{\text{m}} = 81.05 \text{ N.m} \) (Figure 3) and the power factor is also stabilized at rated value \( \cos \varphi_{\text{sh}} = 0.735 \) (Figure 9).

The above-described diagram needs other calculation blocks, if the natural instantaneous values of the voltages and currents are needed. In Figure 2, the stator-oriented model (related to the stator-fixed reference frame) is presented. There is observable, the number of calculation block is more, but the supply needs "natural" two-phase stator- and rotor-voltage variables and also the obtained stator-current is expressed by "natural" two-phase components. In such a case the power factor can be calculated by means of a common Coordinate Transformation (CooT) block characterized by the two-phase rotational operator (symbolized by matrix \( [D] \)) and using it for angle subtraction based on trigonometrical functions [4].

For validation of the machine parameters diagram \( b \) was simulated, realized with a current model for the motor, because it is intended to take into account also the saturation of the iron core [4]. The space phasors were defined with coefficient \( k_{\text{ph}} = 2/3 \), that means the module of the vectors corresponds to the peak value of the sine wave quantities. The mechanical load has reactive character, i.e. its sing is depending on the motor speed.

The simulation results are presented in Figures 3...14. The motor was started straightforward by free acceleration, running under rated feeding conditions (\( U_{\text{IN}} \) and \( f_{\text{N}} \)). Initially it is not loaded. That means on the shaft only the internal torque \( M_{\text{sh}} = 8.1 \text{ N.m} \) is considered, corresponding to the motor mechanical efficiency. Then, after 0.3 sec, the motor was step-loaded with the rated shaft torque \( M_{\text{shN}} = 72.95 \text{ N.m} \). At last the motor is achieving the rated speed \( \Omega_{\text{shN}} = 301 \text{ rad/s} \) (Figure 4) and the sine-wave steady-state operation mode (Figure 11 and 13). It realizes the rated peak values of the rotor current \( I_{\text{r}} = 16.1 \text{ A} \) (Figure 5), stator current \( I_{\text{shN}} = 19.8 \text{ A} \) (Figure 6), stator flux \( \Psi_{\text{N}} = 0.84 \text{ Wb} \) (Figure 7) and resultant stator flux \( \Psi_{\text{z}} = 0.97 \text{ Wb} \) (Figure 8). The electromagnetic torque after the transients becomes equal to the sum \( M_{\text{sh}} + M_{\text{m}} = 81.05 \text{ N.m} \) (Figure 3) and the power factor is also stabilized at rated value \( \cos \varphi_{\text{sh}} = 0.735 \) (Figure 9).

The free acceleration transient mechanical (torque-speed) characteristic is shown in Figure 10. It is based on time-diagrams from Figure 12 and 13 present the space-phasor diagrams of the stator-current and rotor flux related to the stator-fixed reference frame during 0.6 sec for the above described processes.

6. CONCLUSIONS

The approach of the motor electrical and magnetical parameters and quantities was made using the steady-state equivalent circuit, but the validation of them is realized by simulation based on the dynamic model according to the general equations of the induction motor. It was started and loaded under rated conditions and it achieves the rated steady-state operation mode.
Fig. 3. Electromagnetic motor torque.

Fig. 4. Rotor electrical angular speed.

Fig. 5. Module of the rotor-current space phasor (peak value).

Fig. 6. Module of the stator-current space phasor (peak value).

Fig. 7. Module of the rotor-flux space phasor (peak value).

Fig. 8. Module of the stator-flux space phasor (peak value).
The proposed method is dedicated for regular, MS and PhD students, and also for research engineers in the domain of the controlled induction machine drives and/or in automation. It is recommended as a first step of parameter identification in order to make possible the simulation of the field-controlled motors in a very short time.
The method permits to calculate the voltage-, current- and flux-controlled steady-state mechanical characteristics of the induction motor required for frequency-converter fed AC drives.

The presented procedure can be adapted also for wound-rotor induction machines.

7. REFERENCES


